Ideal Gas Law

$$PV = NkT \quad (N = \text{number of molecules})$$

$k = 1.38 \times 10^{-23}$ J/K **Boltzmann's constant**

$$N = nN_A$$

$$PV = nN_A kT$$

$$R = N_A k = 8.31 \ \frac{\text{J/K}}{\text{mol}}$$

$$PV = nRT \quad (n = \text{number of moles})$$

Many problems deal with the changing pressure, volume, and temperature in a gas with a constant number of molecules (and a constant number of moles). In such problems, it is often easiest to write the ideal gas law as follows:

$$nR = \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

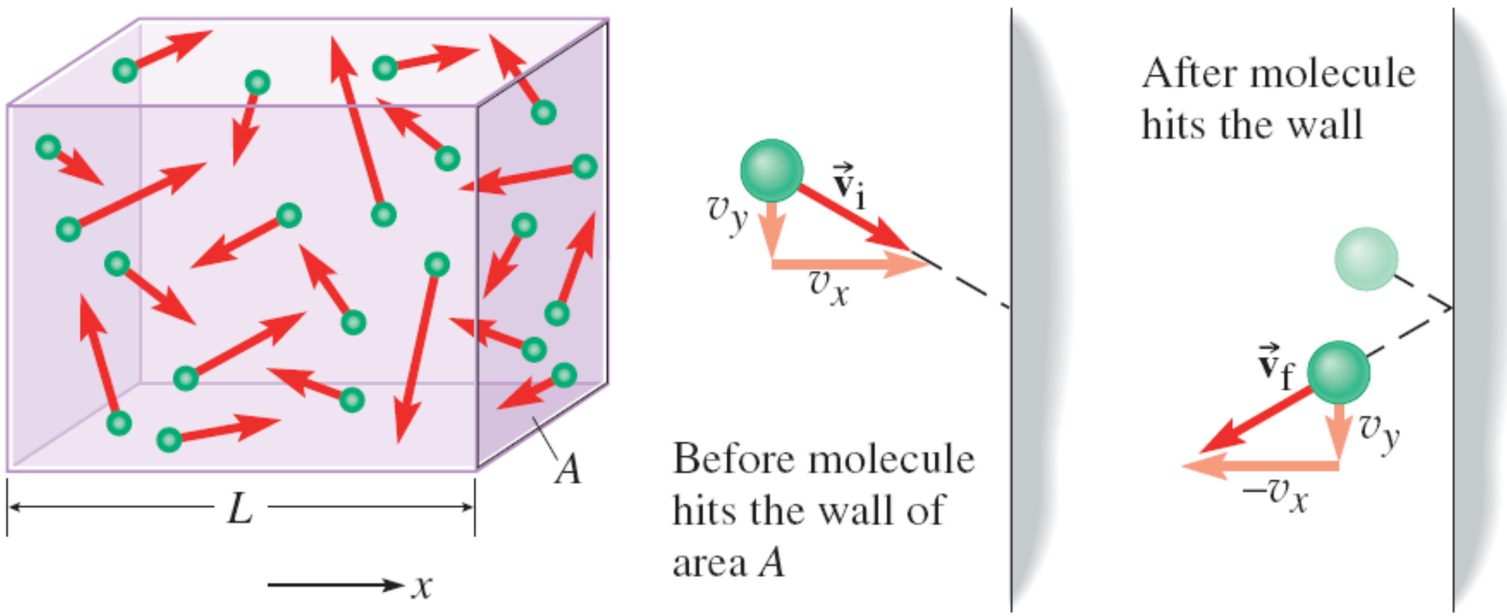
KINETIC THEORY OF THE IDEAL GAS

The **ideal gas** is a simplified model of a dilute gas in which we think of the molecules as point-like particles that move *independently* in free space with no interactions except for elastic collisions.

This simplified model is a good approximation for many gases under ordinary conditions.

Many properties of gases can be understood from this model; the microscopic theory based on it is called the **kinetic theory** of the ideal gas.

Microscopic Basis of Pressure



KINETIC THEORY OF THE IDEAL GAS

$$\Delta p_x = 2m|v_x|$$

$$\Delta t = 2\,\frac{L}{|v_x|}$$

$$F_{\mathrm{av},x} = \frac{\Delta p_x}{\Delta t} = \frac{2m|v_x|}{2L/|v_x|} = \frac{m|v_x|^2}{L} = \frac{mv_x^2}{L}$$

$$F = N\,\langle F_{\mathrm{av}}\rangle = \frac{Nm}{L}\,\langle v_x^2\rangle$$

$$P = \frac{F}{A} = \frac{Nm}{AL}\,\langle v_x^2\rangle$$

$$P = \frac{Nm}{V}\,\langle v_x^2\rangle$$

KINETIC THEORY OF THE IDEAL GAS

$$\langle K_{tr} \rangle = \frac{1}{2} m \langle v^2 \rangle$$

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$\langle v_x^2 \rangle = \frac{1}{3} \langle v^2 \rangle$$

$$m\langle v_x^2 \rangle = \frac{1}{3} m\langle v^2 \rangle = \frac{2}{3} \langle K_{tr} \rangle$$

$$P = \frac{2}{3} \frac{N\langle K_{tr} \rangle}{V} = \frac{2}{3} \frac{N}{V} \langle K_{tr} \rangle$$

Temperature and Translational Kinetic Energy

$$P = \frac{2}{3}\frac{N}{V}\langle K_{tr} \rangle$$

$$\langle K_{tr} \rangle = \frac{3}{2}\frac{PV}{N}$$

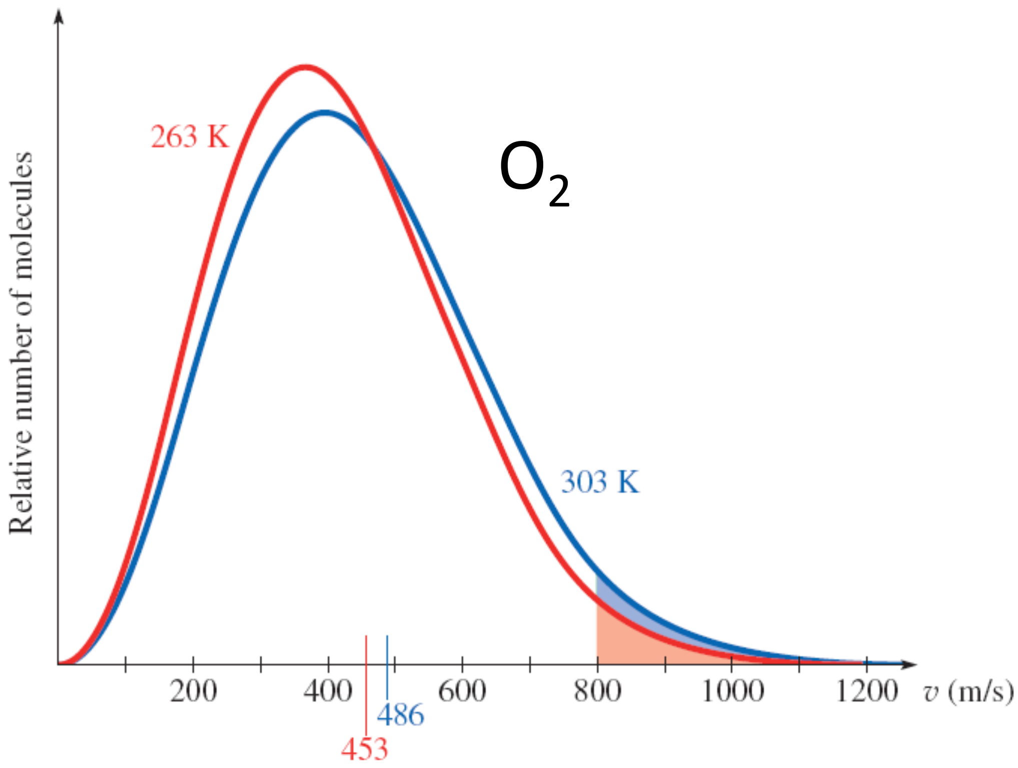
$$PV = NkT$$

$$\frac{PV}{N} = kT$$

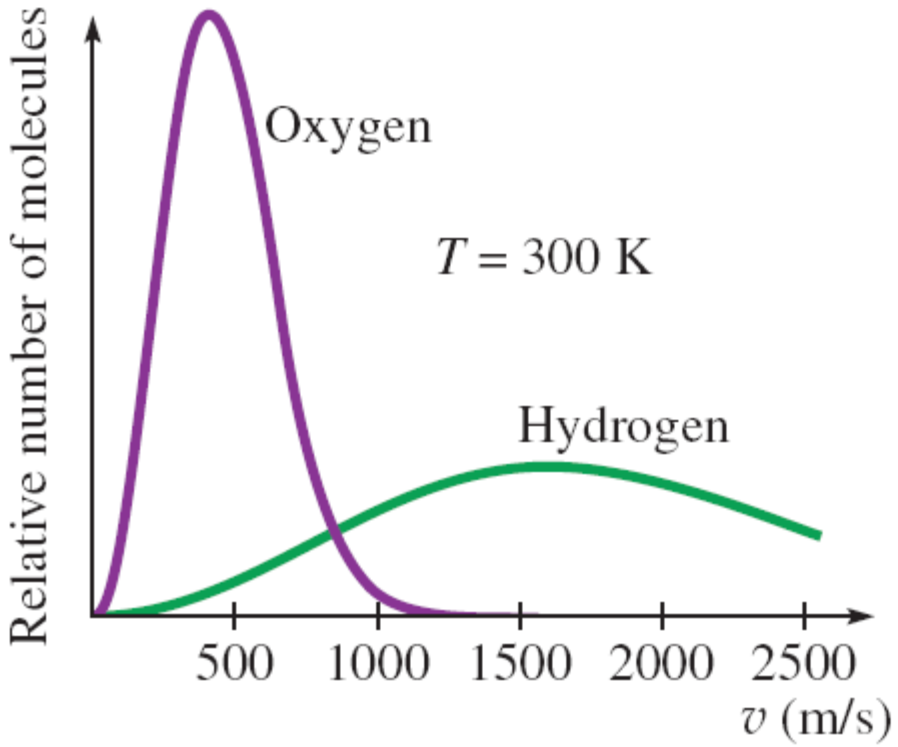
$$\langle K_{tr} \rangle = \frac{3}{2} kT$$

Maxwell-Boltzmann Distribution

Collisions keep the kinetic energy distributed among the gas molecules *in the most disordered way possible* , which is the Maxwell-Boltzmann distribution.



Maxwell-Boltzmann Distribution



RMS Speed

The speed of a gas molecule that has the average kinetic energy is called the **rms** (root mean square) **speed** .

The rms speed is *not* the same as the average speed. Instead, the rms speed is the square *root* of the *mean* (average) of the speed *squared* .

$$v_{\text{rms}} = \sqrt{\langle v^2 \rangle}$$

Example problem: Find the average translational kinetic energy and the rms speed of the O_2 molecules in air at room temperature (20°C).

Strategy

The average translational kinetic energy depends only on temperature. We must remember to use absolute temperature.

The rms speed is the speed of a molecule that has the average kinetic energy.

Solution

$$20°\text{C} + 273\ \text{K} = 293\ \text{K}$$

$$\langle K_{tr} \rangle = \tfrac{3}{2} kT$$

$$= 1.50 \times 1.38 \times 10^{-23}\ \text{J/K} \times 293\ \text{K}$$

$$= 6.07 \times 10^{-21}\ \text{J}$$

$$32.0\ \text{u} \times 1.66 \times 10^{-27}\ \text{kg/u} = 5.31 \times 10^{-26}\ \text{kg}$$

$$\langle K_{tr} \rangle = \tfrac{1}{2} m v_{rms}^2$$

$$v_{rms} = \sqrt{\frac{2\langle K_{tr} \rangle}{m}} = \sqrt{\frac{2 \times 6.07 \times 10^{-21}\ \text{J}}{5.31 \times 10^{-26}\ \text{kg}}} = 478\ \text{m/s}$$