Physics 576: Introduction to Solid State Physics

Spring 2016

Homework 3

(1) *Simon Book Problem 9.3 on vibrations of a one-dimensional monatomic harmonic chain*—Generalize a one-dimensional monatomic harmonic chain with nearest neighbor springs to also include springs between second nearest neighbors. Let the spring constant between the nearest neighbors to be κ_1 and the spring constant between the second nearest neighbors to be κ_2. The mass of each atom is m.

(a) Calculate the dispersion relation ($\omega(k)$) for this model.

(b) Determine the sound wave velocity. Show the group velocity vanishes at the Brillouin zone boundary.

(2) *Simon Book Problem 9.4 on vibrations of a one-dimensional monatomic harmonic chain*—In the dispersion relation of the monatomic harmonic chain (with only nearest neighbors), there is a maximum possible frequency of oscillation ω_{max}. If a vibration with frequency $\omega > \omega_{max}$ is forced on the chain by driving it, the "wave" will not propagate along the chain, but rather will decay as one moves away from the point where the ocsilation is imposed, i.e. the wave is evanescent. With $\omega > \omega_{max}$, solve the dispersion relation for a complex k to determine the decay length of the evanescent wave. What happens to this length as $\omega \to \omega_{max}$?

(3) *Simon Book Problem 10.1: Vibrations of a one-dimensional diatomic chain*—(a) Consider a one-dimensional diatomic harmonic chain where the unit cell is of length a and each unit cell contains one atom of mass m_1 and one atom of mass m_2 connected together by springs with spring constant κ. Each unit cell is also connected together by a spring with spring constant κ.

(a) Derive the dispersion relation for this system.

(b) Determine the frequencies of the acoustic and optical modes at $k = 0$ as well as at the Brillouin zone boundary.

(c) Sketch the dispersion relation in both the reduced and extended zone scheme.

(d) What happens when $m_1 = m_2$?

Homework 3 is due at 2PM Friday, February 19, in my mailbox in PB201